Dark coupling: cosmological implications of interacting dark energy and dark matter fluids

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Abstract. In this study, cosmological models are considered where dark matter and dark energy are coupled and may have non-gravitational interactions with each other. These dark energy couplings are introduced to potentially alleviate both the Hubble tension and the cosmic coincidence problem (regarding the current observed ratio of dark matter to dark energy today). Assuming two different linear dark energy couplings, the conservation and Friedmann equations are used to predict how these couplings affect crucial events in the expansion history of the universe. These events include the big bang and cosmic acceleration, as well as the radiation-matter and matter-dark energy equality. These results are compared with the standard uncoupled ΛCDM model where dark energy is assumed to be a cosmological constant. Cosmological parameters for this study are obtained from Type-Ia Supernovae data using a previously developed Markov Chain Monte-Carlo (MCMC) simulation.

1. Problems with the ΛCDM model

The expansion of the universe has thus far been well described by the ΛCDM model, where the energy budget of the universe is divided between $\approx 5\%$ baryonic matter (standard model particles), $\approx 25\%$ non-baryonic cold dark matter (which keeps galaxies from flying apart) and $\approx 70\%$ dark energy in the form of the cosmological constant Λ (which explains late-time accelerated expansion). This model has proven to be very successful [1], but problems with the ΛCDM model remain, which include:

The Cosmological Constant Problem or vacuum catastrophe, which refers to the measured energy density of the vacuum being over 120 orders of magnitude smaller than the theoretical prediction. This has been referred to as the worst prediction in the history of physics and casts doubt on dark energy being a cosmological constant, motivating research into alternative dark energy models [2].

The Cosmic Coincidence Problem, which alludes to the dark matter and dark energy densities having the same order of magnitude at the present moment of cosmic history, while differing with many orders of magnitude in the past and predicted future [3].

The Hubble Tension, which concerns the 4.4σ level difference between values of the Hubble constant H_0 as measured from the Cosmic Microwave Background (CMB) versus the value obtained from Type Ia Supernovae using a calibrated local distance ladder [4].

2. Dark coupling models

These problems motivate research beyond the ΛCDM model. One possible approach is to investigate cosmological models in which there are non-gravitational interactions between the dark sectors of the universe. This allows the two dark sectors to exchange energy (and/or momentum) while dark matter and dark energy are not separately conserved, but the energy (and

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momentum) of the total dark sector is conserved. This coupling between dark matter and dark energy modifies the continuity equations into [4, 5, 6, 7]:

$$\dot{\rho}_{\rm dm} + 3H\rho_{\rm dm} = Q \qquad ; \qquad \dot{\rho}_{\rm de} + 3H\rho_{\rm de}(1+\omega) = -Q \ , \tag{1}$$

where $H = (\dot{a}/a)$ is the Hubble parameter, with a the scale factor which denotes the relative size of the universe. $\rho_{\rm dm/de}$ is the dark matter/dark energy density, ω is the equation of state of dark energy ($\omega_{\rm dm} = 0$ since dark matter is assumed to be pressureless) and Q is the rate of energy exchange, which defines the direction of energy flow between the dark sectors such that:

$$Q = \begin{cases} > 0 & \text{Dark Energy} \rightarrow \text{Dark Matter} \\ < 0 & \text{Dark Matter} \rightarrow \text{Dark Energy} \\ = 0 & \text{No interaction } (\Lambda\text{CDM case}) \end{cases} \tag{2}$$

The behaviour of coupled models may be understood by how the interaction affects the effective equations of state, relative to the uncoupled background equations ($Q = 0$) in (1) such that:

$$\omega_{\rm dm}^{\rm eff} = -\frac{Q}{3H\rho_{\rm dm}} \qquad ; \qquad \omega_{\rm de}^{\rm eff} = \omega_{\rm de} + \frac{Q}{3H\rho_{\rm de}} \ . \tag{3}$$

Thus, the effects of an interaction may be understood to imply that if:

$$Q > 0 \rightarrow \begin{cases} \omega_{\rm dm}^{\rm eff} < 0 & \text{Dark matter redshifts slower (less DM in past)} \\ \omega_{\rm de}^{\rm eff} > \omega_{\rm de} & \text{Dark energy has less accelerating pressure (older universe)} \end{cases}$$

$$Q < 0 \rightarrow \begin{cases} \omega_{\rm dm}^{\rm eff} > 0 & \text{Dark matter redshifts faster (more DM in past)} \\ \omega_{\rm de}^{\rm eff} < \omega_{\rm de} & \text{Dark energy has more accelerating pressure (younger universe)} \end{cases}$$

When $Q = 0$, the effective equations of state reduce back to the case for the ΛCDM model, where dark matter is pressureless ($\omega_{\rm dm} = 0$) and dark energy has a constant negative pressure.

Since there is currently no fundamental theory for these couplings, they are purely phenomenological and must be tested against observations. Two models will be considered which have interactions proportional to the Hubble parameter [4, 5, 6, 7]. Solving the conservation equations (1) for both models shows how the energy densities evolve, such that:

<u>Model 1: $Q_1 = \delta H\rho_{\rm dm}$</u>

$$\rho_{\rm dm} = \rho_{(\rm dm,0)} a^{(\delta-3)} \tag{4}$$

$$\rho_{\rm de} = \rho_{(\rm de,0)} a^{-3(1+\omega_{\rm de})} + \rho_{(\rm dm,0)} \frac{\delta}{\delta+3\omega} \left[a^{-3\omega} - a^{\delta}\right] a^{-3}, \tag{5}$$

with $\left(0 < \delta < -\frac{3\omega}{(1+r_0)}\right)$ to ensure $\rho_{\rm dm/de} > 0$ throughout evolution.

<u>Model 2: $Q_2 = \delta H\rho_{\rm de}$</u>

$$\rho_{\rm dm} = \rho_{(\rm dm,0)} a^{-3} + \rho_{(\rm de,0)} \frac{\delta}{\delta+3\omega} \left[1 - a^{-(\delta+3\omega)}\right] a^{-3} \tag{6}$$

$$\rho_{\rm de} = \rho_{(\rm de,0)} a^{-(\delta+3\omega+3)}, \tag{7}$$

with $\left(0 < \delta < -\frac{3\omega}{(1+1/r_0)}\right)$ to ensure $\rho_{\rm dm/de} > 0$ throughout evolution.

Here $r_0 = (\rho_{(\mathrm{dm},0)}/\rho_{(\mathrm{de},0)})$ is the ratio of dark matter to dark energy today; and δ is a dimensionless coupling constant which determines the strength of the interaction between dark matter and dark energy. Furthermore, when $Q = 0$, both models reduce to the ΛCDM case where $\rho_{\mathrm{dm}} \propto a^{-3}$ and $\rho_{\mathrm{de}} = \text{constant}$.

Negative couplings ($\delta < 0$) are often used in the literature for these models [4, 6, 7], which are problematic since this leads to negative energy densities in the future. Therefore it is very important to take note of these positive energy density conditions for δ, as they ensure that the energy density for both dark matter and dark energy is not only positive for the past expansion history, but for the future as well.

3. Cosmological parameters

The present cosmological parameters for these models are obtained from a data set of 359 low and intermediate redshift Type-Ia Supernovae (obtained from the SDSSII/SNLS2 Joint Light-curve Analysis (JLA)). This data is used with a previously developed Markov Chain Monte-Carlo (MCMC) simulation for a flat FRLW universe to obtain cosmological parameters for each model from its corresponding Friedmann equation (15). The contribution of $\Omega_{(\mathrm{rad},0)}$ on the expansion and the MCMC model is negligible, but has been chosen as $\Omega_{(\mathrm{rad},0)} = 9 \times 10^{-5}$ =9e-5 (notation used throughout) for further calculations [1]. Details of this MCMC model may be found in [8, 9].

The limit $\omega > -1$ has been imposed for all models, while applying the positive energy conditions $(0 < \delta < -\frac{3\omega}{(1+r_0)})$ and $(0 < \delta < -\frac{3\omega}{(1+1/r_0)})$ for Q_1 and Q_2 respectively. The priors of these parameters are the results from the ΛCDM case. This gives the following results:

Table 1: Cosmological parameters from type Ia Supernovae

Model	$\Omega_{(\mathrm{dm},0)}$	$\Omega_{(\mathrm{bm},0)}$	H_0	ω	δ
ΛCDM	$0.213^{+0.037}_{-0.037}$	$0.055^{+0.031}_{-0.030}$	$69.7^{+0.5}_{-0.5}$	$-1.000^{+0.000}_{-0.000}$	$0.000^{+0.000}_{-0.000}$
$Q_1 = \delta H \rho_{\mathrm{dm}}$	$0.234^{+0.036}_{-0.024}$	$0.043^{+0.022}_{-0.016}$	$68.0^{+0.9}_{-0.9}$	$-0.949^{+0.057}_{-0.036}$	$0.296^{+0.146}_{-0.184}$
$Q_2 = \delta H \rho_{\mathrm{de}}$	$0.232^{+0.031}_{-0.022}$	$0.044^{+0.021}_{-0.017}$	$69.4^{+0.5}_{-0.5}$	$-0.948^{+0.059}_{-0.037}$	$0.257^{+0.161}_{-0.167}$

with $\Omega_{(\mathrm{de},0)} = 1 - \Omega_{(\mathrm{dm},0)} - \Omega_{(\mathrm{bm},0)}$ since a spatially flat universe is assumed. Here it can be seen that H_0 is slightly lower and closer to the CMB value for both Q_1 and Q_2, which slightly alleviates the Hubble Tension [4]. It should be noted that since the conditions to avoid early time instabilities [6] have not yet been considered, these results should be taken as preliminary.

4. Evolution of energy densities

Universe models will be considered which contain radiation (rad), baryons (bm), dark matter (dm) and dark energy (de). In order to avoid fifth force constraints, it is assumed that radiation and baryons are separately conserved and uncoupled [5] such that:

$$\dot{\rho}_{\mathrm{rad}} + 3H\rho_{\mathrm{rad}}(1 + \omega_{\mathrm{rad}}) = 0 \qquad ; \qquad \dot{\rho}_{\mathrm{bm}} + 3H\rho_{\mathrm{bm}}(1 + \omega_{\mathrm{bm}}) = 0 \,, \tag{8}$$

where $\omega_{\mathrm{rad}} = 1/3$; $\omega_{\mathrm{bm}} = 0$. Solving these equations, it is found that radiation evolves as $\rho_{\mathrm{rad}} = \rho_{(\mathrm{rad},0)} a^{-4}$ and baryonic matter as $\rho_{\mathrm{bm}} = \rho_{(\mathrm{bm},0)} a^{-3}$. If the density parameter $\Omega = \rho/\rho_c$ (where $\rho_c = (3H^2/8\pi G)$ and G is the universal gravitational constant) is introduced, the corresponding density parameter for each component may be obtained. This may then be expressed in terms of redshift z through the transformation $1/a = (1+z)$. Therefore the various energy densities $\rho_{\rm i}$ are transformed into the following density parameters $\Omega_{\rm i}$:

$$\Omega_{\rm rad} = \frac{H_0^2}{H^2}\Omega_{(\rm rad,0)}(1+z)^4 \tag{9}$$

$$\Omega_{\rm bm} = \frac{H_0^2}{H^2}\Omega_{(\rm bm,0)}(1+z)^3 \tag{10}$$

$$Q_1 : \Omega_{\rm dm} = \frac{H_0^2}{H^2}\Omega_{(\rm dm,0)}(1+z)^{-(\delta-3)} \tag{11}$$

$$\Omega_{\rm de} = \frac{H_0^2}{H^2}\left[\Omega_{(\rm de,0)}(1+z)^{3(1+\omega)} + \Omega_{(\rm dm,0)}\frac{\delta}{\delta+3\omega}\left[(1+z)^{3\omega} - (1+z)^{-\delta}\right](1+z)^3\right] \tag{12}$$

$$Q_2 : \Omega_{\rm dm} = \frac{H_0^2}{H^2}\left[\Omega_{(\rm dm,0)}(1+z)^3 + \Omega_{(\rm de,0)}\frac{\delta}{\delta+3\omega}\left[1 - (1+z)^{(\delta+3\omega)}\right](1+z)^3\right] \tag{13}$$

$$\Omega_{\rm de} = \frac{H_0^2}{H^2}\Omega_{(\rm de,0)}(1+z)^{(\delta+3\omega+3)}. \tag{14}$$

The ΛCDM equations for $\Omega_{\rm dm}$, $\Omega_{\rm de}$ are obtained by setting the interaction strength $\delta = 0$ in equations (11) - (14). The evolution of these energy densities may be seen in Figure 1:

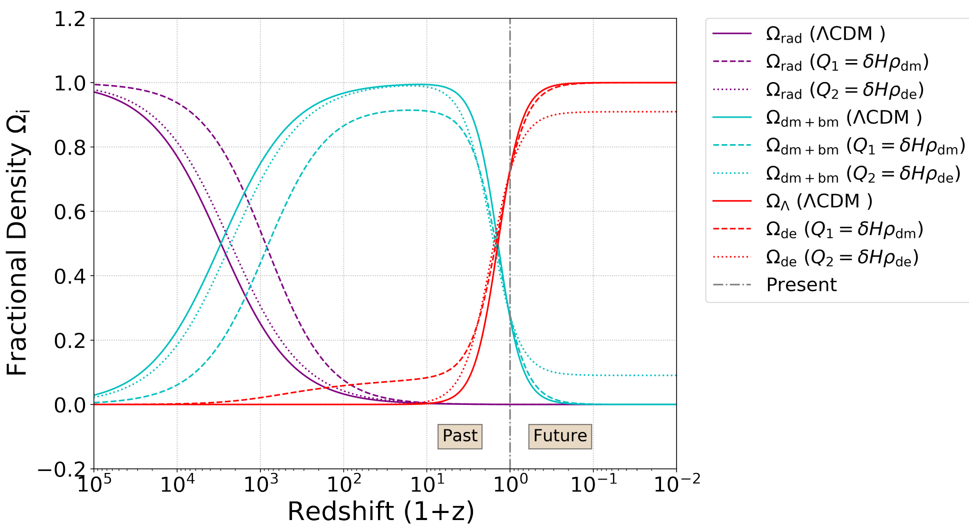

Figure 1: Fractional energy densities vs. redshift.

In all cases, there is an early time radiation domination followed by matter domination, which finally gives way to the the current era of dark energy domination. Here it may be seen that since $\delta > 0 \rightarrow Q > 0$ for both coupled models, that there is less dark matter in the past and that the matter-radiation equality therefore happened later (smaller redshift) in cosmic evolution, while the matter-dark energy equality happens earlier (larger redshift). The energy densities ρ_i (in Joule.m^{-3}) throughout cosmic evolution can be seen in Figure 2. These results are summarised in Tables 2, 3 and 4.

The previously mentioned cosmic coincidence problem may now be addressed by considering how the ratio of dark matter to dark energy $r = (\rho_{\rm dm}/\rho_{\rm de})$ evolves with redshift z in Figure 3. Here it can clearly be seen that for the ΛCDM case, the current value of $r_0 \approx \left(\frac{3}{7}\right)$ seems fine tuned and coincidental in comparison to Q_1 and Q_2, where r converges and becomes constant in the past and the future respectively. Thus, alleviating the cosmic coincidence problem [3].

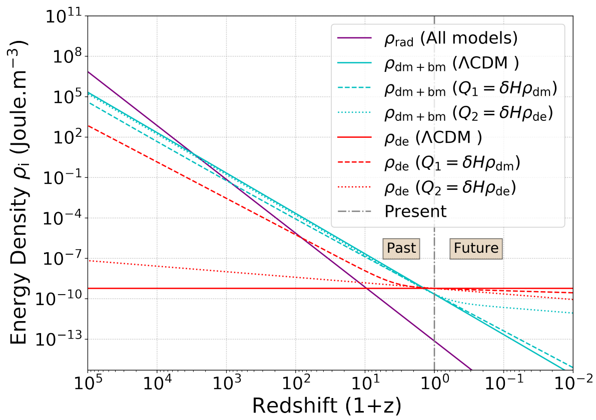

Figure 2: Energy densities vs redshift.

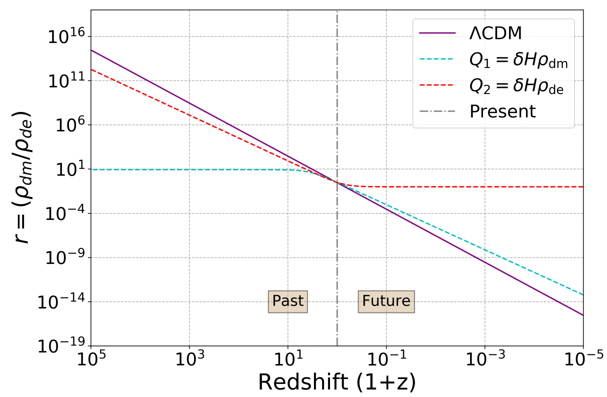

Figure 3: Cosmic Coincidence Problem.

5. Expansion history of universe models

The expansion of these universe models may be described by the Friedmann equation for a flat FLRW universe (15) and the deceleration parameter q (16), with the constituents (9)-(14):

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_{\rm rad} + \rho_{\rm bm} + \rho_{\rm dm} + \rho_{\rm de}\right) \ , \tag{15}$$

$$q = \Omega_{\rm rad} + \frac{1}{2}\left(\Omega_{\rm dm} + \Omega_{\rm bm}\right) + \frac{1}{2}\Omega_{\rm de}\left(1 + 3\omega\right) \ . \tag{16}$$

The Friedmann equation may be numerically integrated, which alongside the deceleration parameter q yields the total expansion histories of the universe models:

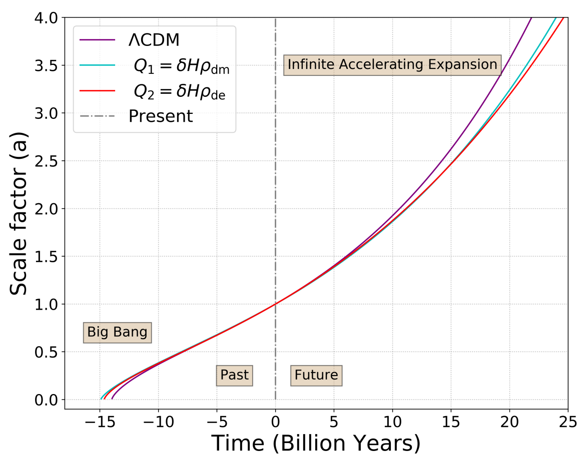

Figure 4: Expansion history of universe models.

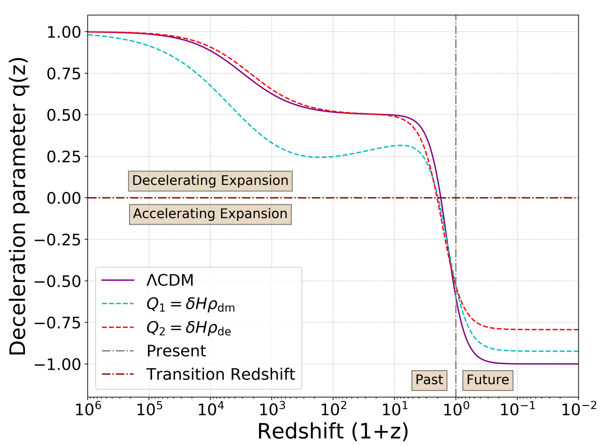

Figure 5: Deceleration parameter vs redshift.

These models all start with a big bang singularity (at $a = 0$) which leads to a period of decelerating expansion during the radiation and matter dominating epochs (the latter starting when $\rho_{\rm rad} = \rho_{\rm dm+bm}$) , followed by an infinite accelerating expansion (starting at $q = 0$ when $\rho_{\rm dm+bm} \approx 2\rho_{\rm de}$) and a final era of dark energy domination (starting when $\rho_{\rm dm+bm} = \rho_{\rm de}$). Tables 2, 3 and 4 show the redshift z, time and energy densities ($\rho_{\rm rad}, \rho_{\rm dm+bm}$ and $\rho_{\rm de}$) at the start of these crucial events in cosmic history for each of the models. It should be noted that cosmological parameters from Table 1 are used for the ΛCDM model instead of Planck CMB parameters [1].

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Table 2: ΛCDM (Supernovae data from [8])

Event	Redshift z	Time (Gyr)	$\rho_{\rm rad}$	$\rho_{\rm dm+bm}$	$\rho_{\rm de}$ (J/m^3)
Big bang singularity	∞	13.96	∞	∞	∞
Radiation-matter equality	2976	13.63	5.7	5.7	5.9e-10
Cosmic acceleration ($q = 0$)	0.76	6.76	7.0e-13	1.2e-9	5.9e-10
Matter-dark energy equality	0.40	4.32	2.8e-13	5.9e-10	5.9e-10

Table 3: $Q_1 = \delta H \rho_{\rm dm}$

Event	Redshift z	Time (Gyr)	$\rho_{\rm rad}$	$\rho_{\rm dm+bm}$	$\rho_{\rm de}$ (J/m^3)
Big bang singularity	∞	14.90	∞	∞	∞
Radiation-matter equality	831.97	14.90	3.5e-2	3.5e-2	1.7e-3
Cosmic acceleration ($q = 0$)	0.94	7.79	1.0e-12	1.4e-9	7.6e-10
Matter-dark energy equality	0.48	5.07	3.5e-13	6.6e-10	6.6e-10

Table 4: $Q_2 = \delta H \rho_{\rm de}$

Event	Redshift z	Time (Gyr)	$\rho_{\rm rad}$	$\rho_{\rm dm+bm}$	$\rho_{\rm de}$ (J/m^3)
Big bang singularity	∞	14.61	∞	∞	∞
Radiation-matter equality	2266	14.61	1.9	1.9	1.4e-8
Cosmic acceleration ($q = 0$)	1.02	8.05	1.2e-12	1.4e-9	7.8e-10
Matter-dark energy equality	0.57	5.60	4.4e-13	7.0e-10	7.0e-10

6. Conclusions

Interacting dark energy models may alleviate the cosmic coincidence problem by stabilising the ratio of dark matter to dark energy in both the past and future (Figure 3). These models also predict a slightly lower value for H_0, thereby showing potential as a candidate for relieving the Hubble tension (Table 1). This lower H_0 value leads to crucial events in cosmic history occurring longer ago with slightly different conditions relative to the ΛCDM model (Tables 2, 3 and 4). These results only hold for energy flow from dark energy to dark matter ($\delta > 0$) since flow from dark matter to dark energy ($\delta < 0$) causes negative energy densities. Finally, since early time instability conditions [5, 6] and other data constraints from the CMB and large-scale structure [1, 4, 7] have not yet been considered, these results should be seen as preliminary.

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